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TITLE AN OVERVIEW OF RELATIVISTIC HYDRODYNAMICS AS APPLIED TO HEAVY ION REACTIONS

AUTHOR(S) Daniel D. Strottman, T-2, Los Alamos National Laboratory

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MASTER

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

AN OVERVIEW OF RELATIVISTIC HYDRODYNAMICS AS APPLIED TO HEAVY ION REACTIONS

D. Strottman

Theoretical Division
Los Alamos National Laboratory
Los Alamos, N.M. 87545

ABSTRACT

The application of relativistic hydrodynamics as applied to heavy ions is reviewed. Constraints on the nuclear equation of state, as well as the form of the hydrodynamic equations imposed by causality are discussed. Successes (flow, side-splash, scaling) and shortcomings of one-fluid hydrodynamics are reviewed. Models for pion production within hydrodynamics and reasons for disagreement with experiment are assessed. Finally, the motivations for and the implementations of multi-fluid models are presented.

I. INTRODUCTION

Hydrodynamics has long been used for the description of high energy reactions since its introduction by Fermi,[1] Landau,[2] and Pomeranchuk.[3] Its application to high-energy hadron-hadron scattering has enjoyed considerable success,[4] albeit not without a certain amount of controversy.[5] Indeed, it successfully describes $p\bar{p}$ reactions up to 800 GeV of center-of-mass energy[6] if the effects due to so-called "leading particles"--created particles that carry away a certain amount of available energy--are taken into account.

In recent years hydrodynamics has also been applied to heavy ion reactions. The original work[7,8,9] centered on the possible existence and implications of shock waves. Despite arguments by Bertsch[10] that nuclear shock waves could not persist, considerable theoretical and experimental work continued. This work has been justified with the experimental verification of many of the original predictions based upon hydrodynamics. Excellent reviews of this earlier work in non-relativistic hydrodynamics may be found in Stocker *et al*.[11]

With the advent of higher-energy heavy ions, the requirement of relativistic hydrodynamics becomes evident. Since the original calculations of Amsden *et al.*,[12] relativistic calculations have been performed by many other groups.[13,14,15,16] These have been the subject of several reviews.[17,18,19,20] In this paper I shall not attempt to duplicate these reviews. Rather, I shall present a brief overview of the successes of the relativistic hydrodynamical model, its deficiencies and its extensions that overcome some of the original problems. The aim is to provide an introduction and to set the framework for the other contributions on relativistic hydrodynamics in this volume.

The use of hydrodynamics to describe heavy ion reactions is very appealing. Hydrodynamics is essentially a classical picture and as such, easily evokes visual images with which one may describe a reaction. A theoretical justification for its applicability to heavy ion reactions is discussed by Maruhn.[21] It has direct connection with the nuclear equation of state, one of the holy grails of heavy ion reactions.* However, it is not a microscopic model and thus cannot address certain questions of direct interest such as thermalization. Nevertheless, it has proven invaluable in providing predictions and correlating data. It will continue to do so as one advances into the higher energy domain.

II. IMPERFECT FLUIDS AND IMPERFECT THEORIES

The relativistic equations for a fluid may be written in the elegant form

$$\partial_\mu N^\mu = 0 \tag{1}$$

$$\partial_\mu T^{\mu\nu} = 0 \ , \tag{2}$$

where the nucleon current is

$$N^\mu = nu^\mu + v^\mu \ , \tag{3}$$

where n is the nucleon density in the rest frame, u^μ the four-velocity and v^μ the particle-diffusion current which is zero for a perfect fluid. In the limit of a perfect fluid, one has

$$N^\mu = (N, N\vec{v}), \ N = \gamma n \ , \tag{4}$$

and

$$\partial_\mu N^\mu = \frac{\partial N}{\partial t} + \nabla \cdot (N\vec{v}) = 0. \tag{5}$$

This equation ensures baryon number conservation and is familiar from the non-relativistic Euler equations.

The expression for the energy-momentum tensor $T^{\mu\nu}$ is more complicated:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P+\tau)q^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu + \tau^{\mu\nu} \ , \tag{6}$$

$$q^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \ ,$$

and the entropy current is

$$s^\mu = snu^\mu + \frac{q^\mu}{T} \ . \tag{7}$$

In Eq. (6) ε is the rest-frame energy density, P is the pressure, τ contains the bulk viscosity, q^μ is the heat conduction tensor and $\tau^{\mu\nu}$ is the shear viscosity tensor. In the perfect fluid limit, τ, q^μ, and $\tau^{\mu\nu}$ are zero and one has

$$T^{\mu\nu} = (\varepsilon+P)u^\mu u^\nu + Pg^{\mu\nu} \ . \tag{8}$$

Inserting $T^{\mu\nu}$ into Eq. (2) and writing in vector notation results in

$$\frac{\partial \vec{M}}{\partial t} + \nabla (\vec{M}\,\vec{v}) = -\nabla P \tag{9}$$

and

$$\frac{\partial E}{\partial t} + \nabla (E\,\vec{v}) = -\nabla \cdot (\vec{v}\,P) \ , \tag{10}$$

*Indeed, the venue of this institute, Peñiscola, is located not far from Montserrat, the legendary repository of the holy grail and the setting of Wagner's opera "Parsifal.

which are the Euler equations for conservation of momentum and density. Had the viscosity or heat conducton terms been retained, the analogous equations would have been somewhat horrific.

There are two special cases of Eqs. (3) and (6) that warrant comment. Equations for an imperfect, relativistic were published by Eckart[24] and Landau-Lifschitz.[22] In Eckart's formulation, one sets v^μ to zero, in which case the four-velocity of the fluid is equal to that of the particles. In the Landau-Lifschitz form, $q^\mu = 0$: the energy flux is zero in the rest-frame of the fluid.

Summarizing: Eckart: $v^\mu = 0$ velocity follows the particles,

Landau-Lifschitz: $q^\mu = 0$ velocity follows the energy.

Much discussion has been spent in arguing the virtue of one choice over the other. For a perfect fluid (no dissipation), the two are equivalent.

In a series of papers, Hiscock and Lindblom[25,26,27] have examined the structure of these first-order theories. (They are referred to as first-order theories because the expression for the entropy current, Eq. (7), contains only deviations from equilibrium that are of first order.) These conclusions are both simple and disturbing:

(a) All equilibria are unstable in the sense that small perturbations away from equilibria will grow exponentially.

(b) The theories appear to be acausal in that it is possible to transmit signals at velocities exceeding that of light.

The time scales for the instabilities are very short. An estimate of the characteristic time scale given in Ref. 27 (neglecting viscosity) is:

$$\tau \gtrsim \frac{\kappa T}{(\varepsilon c^2 + P)c^2} . \qquad (11)$$

(A similar selection holds if viscosity is present.) The value of the heat conductivity κ in nuclear matter is poorly known. However, an estimate for κ given by Csernai *et al.*[28] is

$$\kappa \cong 0.015 \frac{c}{\mathrm{fm}^2} ,$$

whereas an estimate[29] obtained from solving the Boltzmann equation results in

$$\kappa \cong \frac{0.045}{T} \frac{c}{\mathrm{fm}^2} ,$$

where T is measured in MeV. Using the latter estimate and inserting into Eq. (10), one obtains

$$\tau \sim 10^{-28}\,\mathrm{s}$$

for energy densities easily obtained in heavy ion reactions. A value for κ obtained by Danielewicz[30] is somewhat different, but does not change the above conclusion. Thus, one may be forgiven for feeling uneasy when using the first-order theories of Eckart or Landau-Lifshitz and including dissipation.

However, in the same series of papers, Hiscock and Lindblom have shown that a second-order theory proposed by Israel and Stewart[31] is both causal and stable. However, it has the disadvantage of being appreciably more complicated. Rather than the five degrees of freedom in the first-order theories, there are now fourteen. The solution of these more general equations has not been done.

The conclusions reached by Hiscock and Lindblom are not without controversy and are not accepted by everyone. It should be noted that the expression for the energy-momentum tensor, $T^{\mu\nu}$, Eq. (6) does not change in the Israel-Stewart theory. Rather, the expression for the entropy current, Eq. (7), is generalized to include terms of second order in deviations from equilibrium. The requirement that entropy not decrease (second law of thermodynamics) then imposes requirements on the form of the current, which in turn imposes conditions on the nature of heat conduction and viscosity in $T^{\mu\nu}$. These conditions are different in the first-order and second-order theories.

Olson and Hiscock[32] have recently pointed out that these constraints also restrict the nature of the nuclear equation of state. The requirement that the adiabatic sound speed, v_s, be subluminal requires the energy per nucleon not rise faster than linear[33] for large density. The consideration[34] of the thermal degrees of freedom imposes further constraints; the thermodynamic constraints derived by Olson-Hiscock are even more restrictive. For example, the Sierk-Nix[35] equation of state, which rises linearly with density for large n produces unstable solutions at 15.2 times normal density, although v_s exceeds one only at 740 n/n_0. Similarly, the Skynne interaction SkM* for which v_s exceeds one[32] at 9.3 n/n_0 has regions of instability already at 5.8 normal density. These values are assuming the thermal degrees of freedom are those of a degenerate Fermi gas. Clearly, more realistic examples should be investigated; most likely this would result in even lower values of allowed densities. Nevertheless, this suggests the utility of looking at the constraints imposed by a relativistically-correct theory.

III. ONE-FLUID HYDRODYNAMICS

When one speaks of hydrodynamics, one invariably means one-fluid hydrodynamics. The non-relativistic calculations are discussed by Maruhn[21] elsewhere in this volume. By one-fluid hydrodynamics, one means the target and projectile are essentially part of the same fluid. Because, in the absence of viscosity, the mean free path of a fluid is zero, this is then a statement that when two nuclei collide, stopping is immediate and equilibration instantaneous. This clearly restricts the domain of applicability of one-fluid hydrodynamics to that region for which the mean free path of an energetic nucleon is appreciably less than the nuclear radius. An estimate of the mean free path is

$$\lambda \sim \frac{1}{\sigma\rho} \; ; \tag{12}$$

for high energies this gives an estimate of about two fm. Several collisions are required for full stopping and it is clear one-fluid hydrodynamics is only an approximation for high energy.

Because this subject is discussed both elsewhere in this volume, as well as in two recent reviews,[17,18] we shall be brief. The triumphs and successes of hydrodynamics will be briefly discussed; its problems and shortcoming will be reviewed, as well as possible future remedies.

III.1 KINETIC ENERGY FLOW AND SIDE-SPLASH

One of the original predictions of hydrodynamics was that of shock waves and the concomitant side-splash. This may be viewed in a classical picture as arising from the collision of two nearly incompressible objects in a non-central collision. Although the magnitude of the deflection is related to the compressibility of the fluid, clearly one expects a qualitative difference between this picture and that of two clouds of gas colliding and in which the particles would pass through each other virtually unscathed. Fig. 1 presents the results of a three-dimensional calculation of ^{20}Ne colliding with ^{238}U; the calculation is in the laboratory reference frame. The deflection of the ^{20}Ne (and the shearing of part of its side) and the presence of a sidewards shock wave are apparent.

Although such effects were sought in early studies, confirmation of these predictions had to await the development of 4π detections. Among the first techniques suggested to quantify this collective motion was kinetic energy flow. It was suggested by Gyulassy *et al.*[36] that one form the 3×3 matrix,

$$K_{\alpha\beta} = \sum_i \frac{p_\alpha^{(i)} p_\beta^{(i)}}{2m_i} \quad . \tag{13}$$

where the sum i runs over all detected particles, α and β denote the three spatial directions, and the p_α are the three-momenta of the particles.

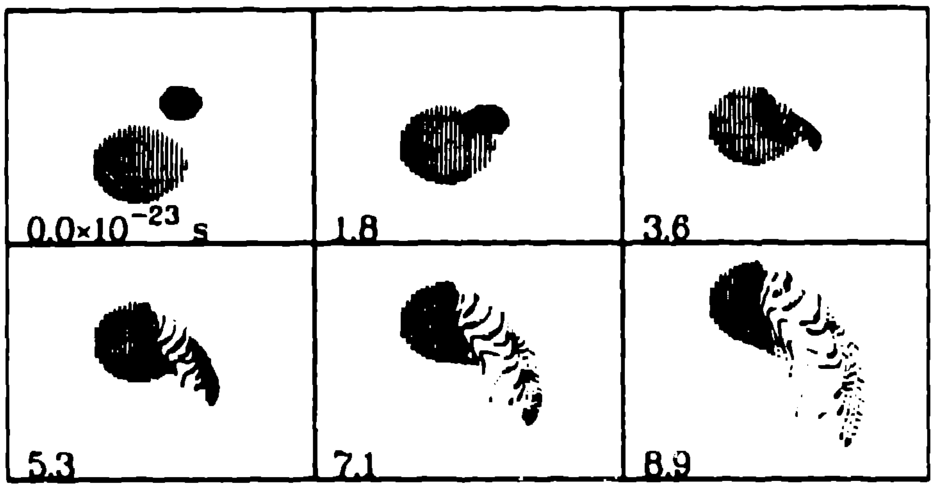

Fig. 1. Time evolution of the nucleon density during the reaction ^{20}Ne on ^{238}U at a bombarding energy of 400 MeV per nucleon. The calculation was performed in the center-of-velocity frame using the particle-in-cell method. In the initial frame each dot represents a column of matter; as the reaction evolves and waves appear, the matter in the column become skewed and the wave visible.

The construction and diagonalization of the 3×3 matrix is equivalent to transforming to a body-fixed axis oriented along the principal axes of a rigid body. The six independent matrix elements of the 3×3 array may be related to the three principal axes and the three Euler angles which one may obtain by diagonalizing the 3×3 matrix. The trace is simply the total non-relativistic kinetic energy. If one labels the three eigenvalues f_i so that $f_1 \geq f_2 \geq f_3$, the polar angle θ_1, of maximum flow is denoted by θ_F.

Two limits immediately show the usefulness of this analysis. Consider two equimass nuclei in a central collision. If they are relatively transparent and they pass through one another, little matter will be pushed into the transverse direction. In this case the flow angle θ_F will be zero since the largest axis f_1 will be along the z-axis. However, if there is complete stopping as in hydrodynamics, the strong pressure will cause matter to be squirted out the sides in the transverse directions and θ_F will be 90°.

The results in Fig. 2 are plotted from one- and two-fluid hydrodynamic calculations. The abscissa is the ratio f_1/f_3. One immediately sees there are large differences between the calculations and hence, a measurement of the flow angle could, in principle, provide information on transparency as well as the equation of state.

Unfortunately, there are difficulties analyzing experimental data with the approach. The first is fluctuations associated with the finite particle number. A suggestion to overcome this inherent problem was made by Danielewicz and Gyulassy[37] but the problem, although mitigated, remains. A second problem is the intrinsic inability to define from experiment a precise impact parameter.

Nevertheless, an analysis by Buchwald *et al.*[38] of flow from hydrodynamic calculations shows good agreement with 400-MeV A data[39] from the plastic ball at the Bevalac, as is apparent in Fig. 3, taken from their paper. This provides strong evidence for the applicability of hydrodynamics at these energies and is a triumph for the predictions made many years before such sidewards flow was seen.

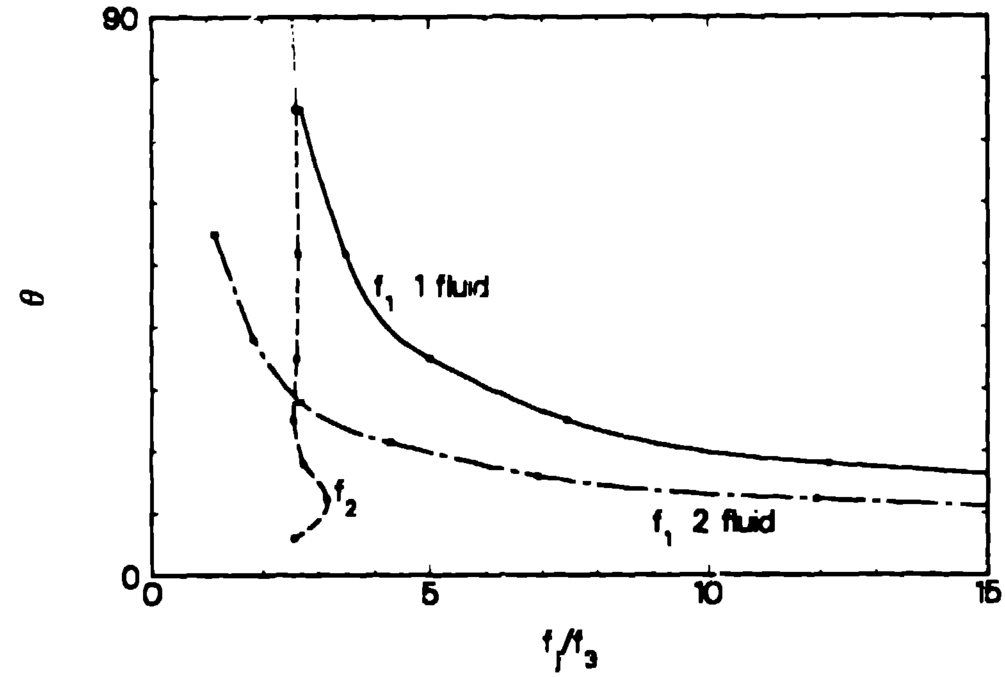

Fig. 2. Results for f_1/f_3 and f_2/f_3, the ratio of eigenvalues of the kinetic energy tensor for both a one-fluid and a two-fluid calculation, from the reaction of mass 40 nuclei colliding at 800 MeV per nucleon. The indicated points represent different impact parameters b in units of the sum of the radii $R_1+R_2 = 2R$; thus, b = 1 represents a grazing collision. The circles represent impact parameters of 0.05, 0.15, 0.25, etc., with the larger angles corresponding to smaller b.

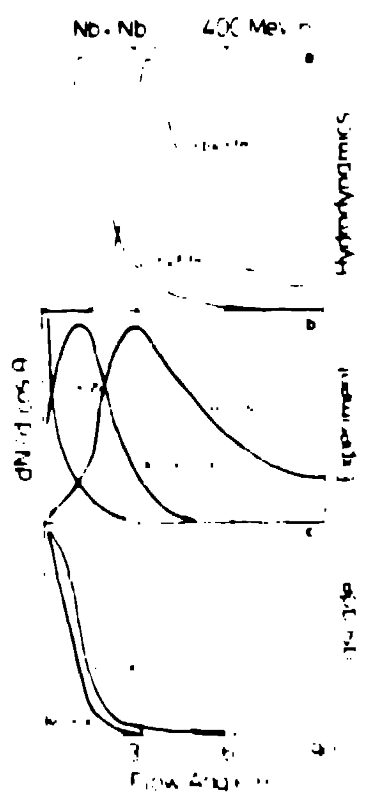

Fig. 3. Distribution of flow angles for the reaction $^{93}Nb+^{93}Nb$ at 400 MeV per nucleon. Results shown are from a hydrodynamic model and cascade model as well as experiment. The figure is from Refs. 17 and 38.

III.2. TRANSVERSE FLOW

To overcome the problems engendered by a finite multiplicity, Danielewicz and Odyniec[40] proposed that a useful measure of flow was the transverse momentum flow, p_x, in the reaction plane. In addition, they proposed a new and successful method of determining the reaction plane from the observed fragments. Their method is now the standard method of comparing predictions with measured flow.

In Fig. 4 are shown results for transverse flow per nucleon, p_x/A, obtained from a two-fluid model for collisions of equi-mass nuclei for several impact parameters. As one might expect, there is a clear dependence on impact parameters. In Fig. 5 are shown results for transverse momentum, p_T/A.

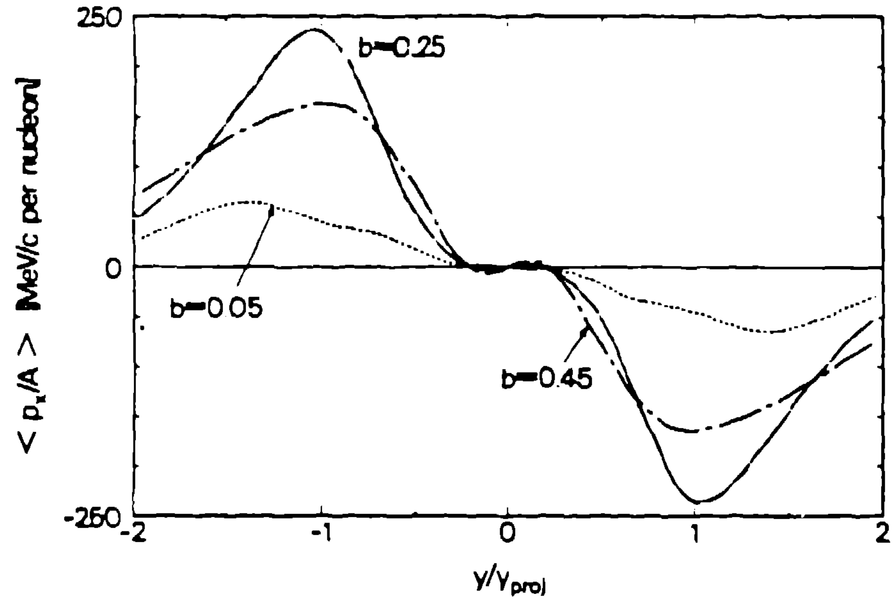

Fig. 4. Results for p_x/A resulting from a collision of mass 40 nucleons at 1.8 GeV per nucleon for impact parameters of 0.05, 0.25, and 0.45, as calculated in the two-fluid model. At this bombarding energy results from a one-fluid model are similar.

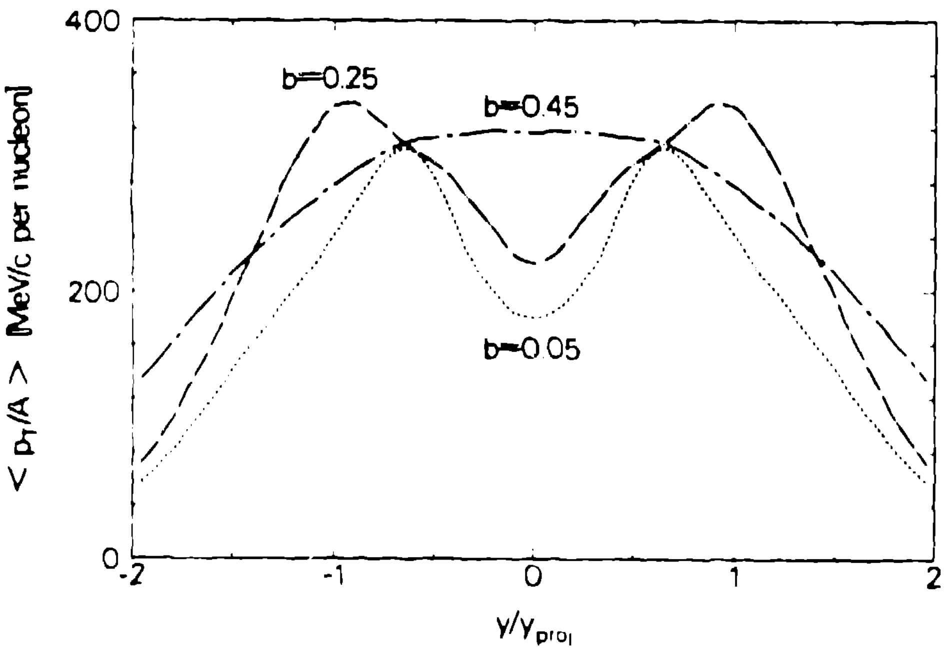

Fig. 5. Results for p_T/A resulting from a collision of mass 40 nucleons at 1.8 GeV per nucleon for three impact parameters, as calculated in the two-fluid model. At this bombarding energy, results from a one-fluid model are similar.

After maximum compression is reached in a collision, the nucleon fluid will expand until hydrodynamics ceases to be valid. The fluid should then break into nucleons or higher mass composites and travel to the detectors. However, this break-up time is not well-defined and many of the calculated observables may depend upon whatever one assumes. Two assumptions are generally used:

1. Global; break-up time is everywhere the same and is chosen by some criteria based, e.g., on the average density or temperature;

2. Cell-by-cell; the time for freeze-out occurs when the matter in each individual cell drops below a prescribed density or temperature.

Figure 6 shows results for four different global break-up times. One may observe there is some dependence on the assumed break-up time, but, fortunately, the dependence is weak. In general, it is found that if the calculation has a sufficient number of cells, once the calculation proceeds past a certain shape, $<p_x/A>$ and $<p_T/A>$ remain essentially unchanged.

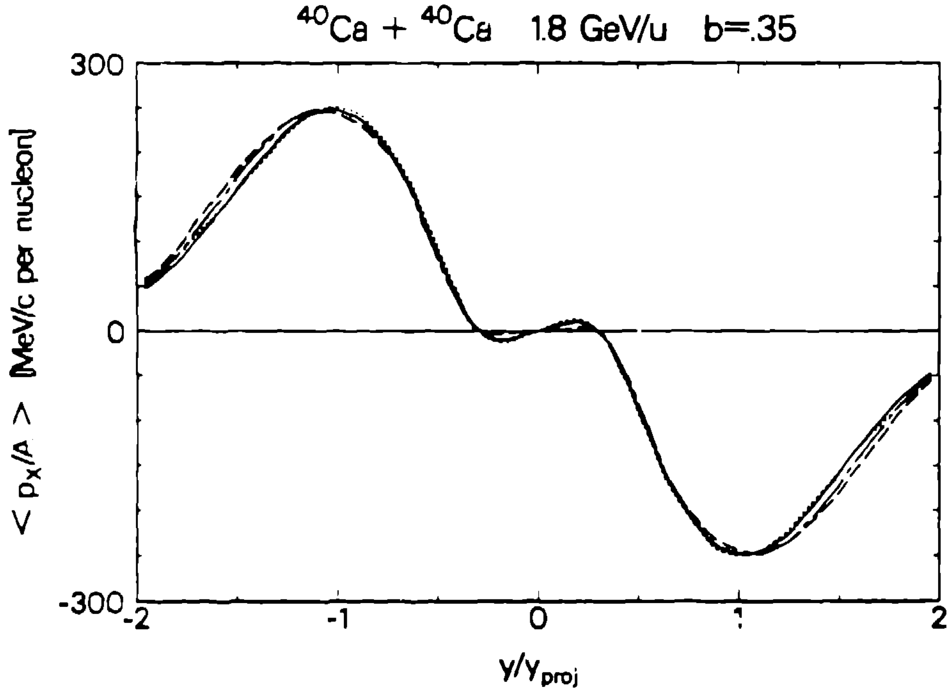

Fig. 6. Average transverse momentum per nucleon, p_x/A, for four assumed break-up times for mass 40 on mass 40 at 1.8 GeV/u. The times in the calculational frame for the four curves are 11.4, 13.6, 15.9, and 18.2 fm/c.

III.3. SCALING

In the absence of dissipation, the Euler equations have only first-order derivatives in both space and time. Thus, if the spatial components are all increased by a factor ε: $\bar{r} \rightarrow \varepsilon\bar{r}$, the solutions to the Euler equation will remain unchanged if the time is scaled by the same amount. The Euler equations do not provide a fundamental scale and hence the results will apply equally well for all nuclei if scaled by $A^{1/3}$. In the non-relativistic regime the results should also scale as $E^{1/2}$.

The addition of dissipation breaks scaling since this inserts terms containing higher derivatives. Thus, one method to look for the effects of viscosity would be to check for violations of scaling. Unfortunately, there are other, physical processes that can lead to violations of scaling. For instance, particles near the surface can escape or the surface regions can freeze-out earlier than the interior regions; these effects need not scale as $A^{1/3}$. As one increases the energy, new physical processes such as pion emission are allowed. Finally, if one encounters a phase transition, scaling would also break down.

Scaling has been examined by Balazs *et al.*[42] and Bonasera and Csernai.[43] Results from the latter analyses are shown in Fig. 7. The small deviations from scaling can be attributed to viscosity, finite mean free path, edge effects, and other mechanisms.

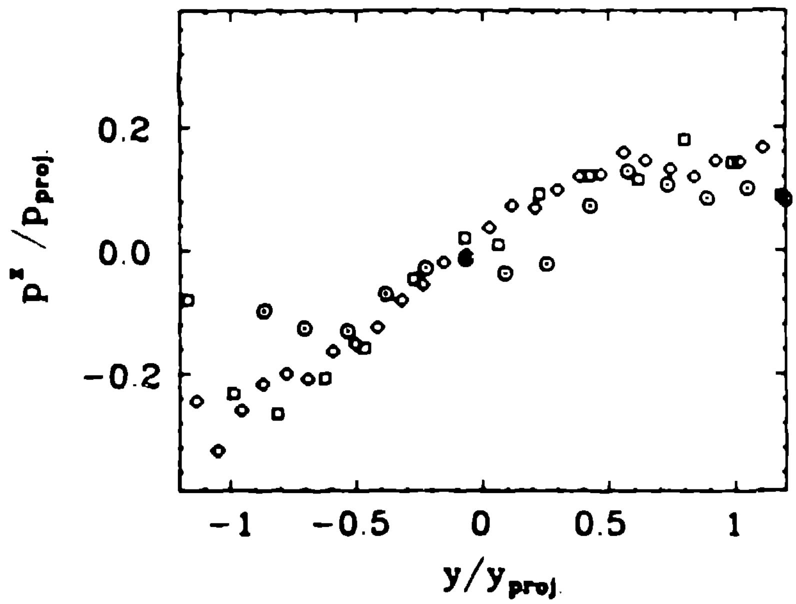

Fig. 7. Scale-invariant transverse momentum plotted as a function of scale-invariant rapidity for three different reactions: Ar + KCl, 1.8 GeV/u (dotted circles); La + La, 0.8 GeV/u (squares) and Nb + Nb, 0.4 GeV/u (diamonds). The figure is taken from Bonasera and Csernai.[43]

IV. MULTI-FLUID FLOW

Multi-fluid flow for the description of heavy ion reactions has been introduced for a variety of reasons. The formulation of the equations to describe a multi-fluid system is formally straightforward and is a generalization of the one-fluid equations. However, the coupling terms that describe the new physics may generate considerable practical difficulties.

The hadron current for an N fluid system is

$$N^{\mu}(x) = \sum_{k=1}^{N} N_k^{\mu}(x) \; ; \tag{14}$$

the sum is over the types of particles allowed, although one generally has $\partial_\mu N^\mu = 0$, the individual baryonic currents may or may not satisfy this conservation law. In the one-fluid model the baryon current was identical to that of the nucleons and $\partial_\mu N^\mu = 0$ simply represented conservation of baryon number. If the number of species can change and, in particular, if there are mesons or anti-particles which can be created and absorbed, the current describing each species clearly need not be conserved.

Similarly, the energy-momentum tensor is

$$T^{\mu\nu}(x) = \sum_{k=1}^{N} T_k^{\mu\nu}(x) \; . \tag{15}$$

Although the total energy and momentum are conserved, that of each individual fluid need not:

$$\partial_\mu T_k^{\mu\nu} = F_k^{\nu}(x) \; . \tag{16}$$

The coupling functions F_k^v determine the new physics and distinguish one model or variant from another. The motivations for introducing multifluids may be grouped into three categories:

1. Inclusion of Transparency. This was the motivation for the original work by Amsden *et al.*[46] on the two-fluid model. Perfect fluid hydrodynamics (no dissipation) necessarily implies a zero mean free path. The use of viscosity allows small, but finite mean free paths. However, at relativistic energies, the nuclear stopping distance may become comparable to, or exceed, the nuclear dimension. At this point it becomes very difficult to justify hydrodynamic calculations that assume $\lambda = 0$.

   These considerations also motivated the work of Danielewicz *et al.*[47] and Ivanov *et al.*[48] within the context of the firestreak model and Csernai and Barz[49] for the quark-gluon plasma. This work was then extended to a two-fluid model and is described by Satarov *et al.*[50] and Mishustin *et al.*[19] as well as in separate contributions to this volume.[51]

2. Thermalization. In the one-fluid model, matter that is compressed or shocked is assumed to thermalize immediately. Although this is clearly not a correct description of a reaction, it may not be a poor assumption at the lower bombarding energies. At higher energies for which the reaction time is comparable with the relaxation time, it is clearly a poor approximation of reality. The introduction of two fluids mitigates the problem but does not explicitly address the issue of thermalization.

   In papers by Csernai *et al.*[28] and Rosenhauer *et al.*[52] the approach to thermalization was studied by introducing a three-fluid model. In addition to the two fluids representing the two colliding nuclei, a third fluid was introduced that described the thermalized fluid. Although these models are an improvement on the one-fluid calculations, the models still have a problem in that if one element of a nucleus becomes part of the third or thermalized fluid, it is forever a part of this fluid. In reality, a system may evolve so that part of a thermalized fluid may become unthermalized. Such a possibility could be incorporated into future calculations by a generalization based on the Boltzmann equation of the coupling terms.

3. Multi-Component Fluids. During the course of a heavy ion reaction, many particles of different species, such as pions, kaons, *etc.*, can be produced. In principal each of these species could be described as a separate fluid, the evolution of which could be governed by hydrodynamics or kinetic theory. The equations for this this for heavy ion reactions were first obtained by Clare[18] and were elaborated by Rosenhauer *et al.*[53] In this latter work the two-fluid and three-fluid models can be obtained as limits--although the merging of the two fluids as in the calculation of Amsden *et al.*[46] is missing. No numerical calculations have been reported within this third approach.

To obtain the equations that describe our two-fluid model, each nucleus is assumed to be a fluid that has the identical properties of the fluid representing the other nucleus. When the two fluids collide they are allowed to exchange energy and momentum at a rate proportional to the relative velocity of the two fluids and to the NN cross section appropriate for that velocity. Thus, the rate of momentum loss of each nucleus is finite and the fluids may interpenetrate. The amount of interpenetration will be small at low energies where σ_{NN} is larger and increases as σ_{NN} decreases. The coupling terms were estimated using arguments from kinetic theory; if one knows the collision rate and the amount of energy and momentum lost in each nucleon-nucleon collision, then the total amount of loss may be found.

The expression for the collision rate is

$$R_{coll} = N_1 N_2 \sigma_{NN} v_{rel} \ ,$$

where N_1 and N_2 are the densities of the two fluids and v_{rel} is the relative velocity of the fluids. This expression must be written in a form appropriate for relativistic transformations, which is done by noting that N_1 and N_2 are the nuclear number densities in the calculational frame

$$N_1 = \gamma_1 n_1 \ , \qquad N_2 = \gamma_2 n_2 \ .$$

σ_{NN} is the Lorentz invariant cross section for an NN reaction, and v_{rel} is a relativistic generalization of the relative velocity

$$v_{rel} = [(v_1 - v_2)^2 - (v_1 \times v_2)^2]^{1/2} \ .$$

The quantity $N_1 N_2 v_{rel} = X n_1 n_2$, where

$$X = \gamma_1 \gamma_2 v_{rel}$$

is the Lorentz-invariant Møller flux factor.[54]

The equations are obtained by deriving them in the rest frame of fluid 2 and then transforming to an arbitrary frame. In the rest frame of fluid 2, $X = \lambda_1 v_1 = p_1/m_1$. The average longitudinal momentum transferred in each collision from fluid 1 to fluid 2 is assumed to

$$\delta M_1 = K(X)\, v_1 \ ,$$

where K is a function of X. The total momentum transfer is then $R_{coll}\, K(X)\, v_1$. Making use of the scalar invariant $u_1 \cdot u_2$, which is $-\gamma_1$ in the rest-frame of fluid 2 (where u_i is the 4-velocity of the i^{th} fluid) one has

$$v_1 = -[u_1 + (u_1 \cdot u_2)\, u_2] / (u_1 \cdot u_2) \ . \qquad (17)$$

When transformed to an arbitrary frame, this will in general contribute to both the spatial and time components.

The average kinetic energy transfer in a collision is assuemd to be

$$\delta E = K' v_1^2 = K'(X/Y)^2\, u_2 \ , \qquad (18)$$

where

$$Y = (u_1 \cdot u_2) = \sqrt{1 + X^2} \ ,$$

X is defined above and $u_2 = (0,1)$ in the rest-frame of fluid 2. The function K' is a function of X. The total energy transfer is $R_{coll} K' v_1^2$. A relation between K and K' is found by transforming the equations to the rest-frame of fluid 1 and requiring that one obtain the same form of the equations as one did in the rest frame of fluid 2,

$$K'(X) = K(X) Y / (1 + Y) \ .$$

After transforming Eqs. (17) and (18) to an arbitrary frame, the Euler equations for fluid 1 are

$$\frac{\partial M_1}{\partial t} + \nabla(v_1 \bullet M_1) = -\nabla P_1 - R_{coll} \frac{K}{Y} (\gamma_1 v_1 - \gamma_2 v_2) \ . \qquad (19)$$

$$\frac{\partial E_1}{\partial t} + \nabla(v_1 E_1) = -\nabla(v_1 P_1) - R_{coll} \frac{K}{Y} (\gamma_1 - \gamma_2) \ . \qquad (20)$$

Equations for fluid 2 are found by interchanging the labels for 1 and 2.

Unlike the one-fluid model, the equations (19) and (20) for the two-fluid model are not scale invariant. The calculated results depend upon the mass of the nuclei involved and this is entirely reasonable. A similar consequence occurs if one uses the Navier-Stokes equations. However, unlike the case of the Navier-Stokes equations which introduce dissipation through higher-order derivatives of velocity (and the ensuing complications), the two-fluid model achieves the same end by eliminating derivatives in the additional terms.

The additional coupling terms in Eqs. (19) and (20) describe the friction between the nuclei entirely in terms of two-body collisions of constituent nucleons. It is assumed that the NN cross section is the free NN cross section and is independent of density (which is probably a poor assumption at large densities) and that the Fermi velocities of the individual nucleons may be ignored. For large relative velocities this is a good approximation but at lower velocities it probably underestimates the coupling. However, at sufficiently low velocities it is irrelevant because the fluids will have merged together.

When the relative velocity of the two fluids is less than the Fermi velocity, it is difficult to continue speaking of two separate and distinct fluids. Thus, at low energies the two-fluid model is assumed to go over to the one-fluid model. This transition is performed in the following manner. When the relative velocity X exceeds an upper value X_u (*e.g.*, twice the Fermi velocity), it is assumed the equations for the two-fluid model remain unchanged. Below a lower limit X_ℓ, (*e.g.*, the Fermi velocity) it is assumed the one-fluid model is valid. In this latter case the one-fluid quantities are related to the two-fluid quantities by

$$N = N_1 + N_2, \qquad E = E_1 + E_2$$

and

$$M = M_1 + M_2 .$$

For values of X intermediate between X_ℓ and X_u, one assumes the values of pressure and velocity for each fluid is obtained from

$$\langle P_i \rangle = \alpha P_i + (1 - \alpha) N_i P / N$$

and

$$\langle v_i \rangle = \alpha v_i + (1 - \alpha) v ,$$

where

$$\alpha = (X - X_\ell) / (X_u - X_\ell), \quad X_\ell \le X \le X_u .$$

For X above X_u, $\alpha = 1$ and for X less than X_ℓ, $\alpha = 0$. Thus, α varies linearly from 0 to 1. In addition, the coupling terms in Eqs. (19) and (20) are also multiplied by α, *i.e.*, R_{coll} becomes $\alpha(X)\,R_{coll}$. Thus, when the fluids have merged there is no further exchange of energy and momentum between the two fluids. The coupling function K(X) is chosen to reproduce the mean longitudinal momentum transfer in a nucleon-nucleon collision.

In Fig. 8 are shown the matter distribution for ^{32}S on ^{208}Pb at 60 GeV per nucleon. It may be observed that there is partial transparency: some of the matter originally belonging to the projectile has passed through the Pb target. However, this projectile matter has a velocity appreciably less than its original velocity, in keeping with the choice of coupling function K(X), which should provide for a mean rapidity loss of two units.

The work of Busza and Goldhaber[55] analyses inclusive cross-section data[56] for the reaction $p + A \rightarrow p + X$ for which the incident proton energy is 100 GeV. They conclude the mean rapidity loss of a 100-GeV proton traversing the diameter of a heavy nucleus is approximately 2.4 units. A coupling function that reproduces this rapidity loss is

$$K(X) = \frac{1}{4} m_N \left(6 - \frac{5}{1+X^2}\right) .$$

It is the coupling function that was used in the calculation shown in this paper.

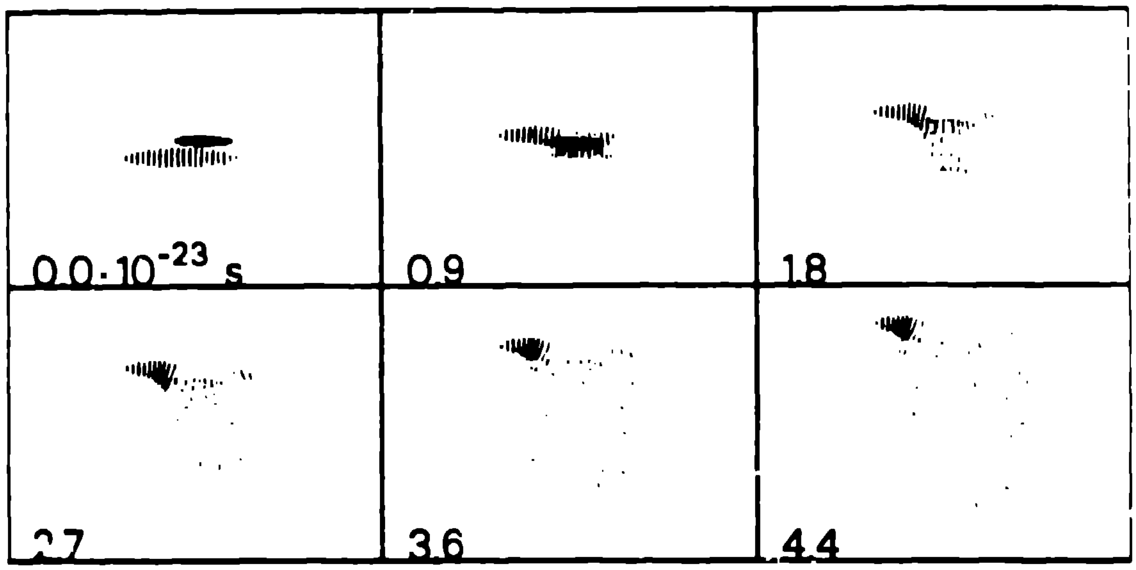

Fig. 8 Matter distributions calculated for the collision of ^{32}S on ^{208}Pb at 60 GeV per nucleon. The calculation is in the center-of-velocity frame.

V. PIONS

Being the lightest meson, the pion is produced more abundantly than any other meson. It couples strongly to the nucleon, true pion absorption is an important but not completely understood process.[57] Attempts to link pion production in heavy ion reactions to the nuclear equation of state[58,59] have been made. This linkage has failed, in part, because medium effects known to be important have not previously been included in the analyses of pion production, nor have many-body effects as as three- and four-body absorption (see Fig. 9). Although these mechanisms account for perhaps 30% and 10%, respectively, of true pion absorption at normal density,[60] their density dependence , ρ^3 and ρ^4, respectively, and hence, may dominate at high densities. The explicit inclusion of such effects has not been attempted.

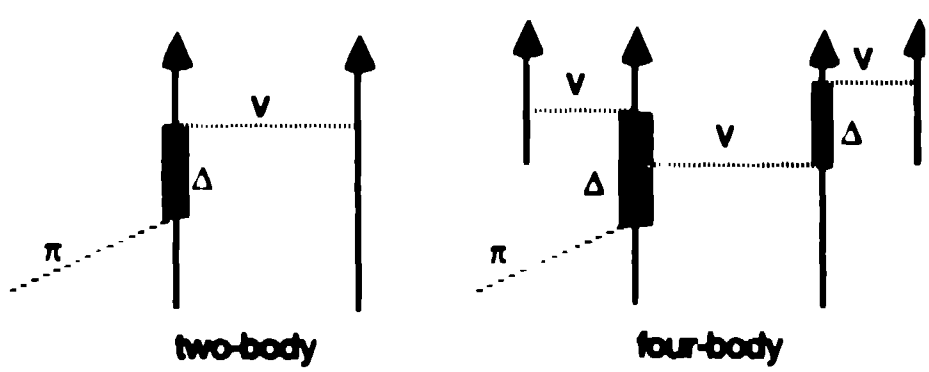

Fig. 9 Diagram illustrations of two- and four-body true pion absorption

However, corrections due to medium effects have been studied in some models. Medium corrections take into account interactions between the pions and hadrons in the nuclear medium and have the effect of altering the pion's properties and interactions. Migdal[61] suggested these corrections might give rise to a pion condensate in the nucleus. After much theoretical and experimental work, it appears a condensate or incipient condensate does not exist at normal nuclear density. However, it is not to be precluded that the medium corrections can give rise to large, collective effects at higher densities reached during a heavy ion reaction with interesting consequences.[62]

At moderate energies of a few hundred MeV per nucleon, the strong interest in pion production stems in part from the suggestion of Stock *et al.*[58] that pion yields from heavy ion reactions would yield information about the nuclear equation of state. This was motivated by the realization that most early models of heavy ion reactions yielded too many pions compared to experiment; these early calculations--performed with either thermodynamic[63] or intranuclear cascade models[64]--did not include any compressional energy. Hence, there was too much thermal energy in the expanding system; this energy was presumed to go into pion production. If one arbitrarily added sufficient compressional energy to reduce the excess number of pions until there was agreement with experiment, the suggestion was that one had a measure of the compressional energy of the dense fireball resulting from the collision of the heavy ions. [This argument seemed to also be given credence by hydrodynamical calculations[65,66] which do include compressional energy in a natural way. These calculations predict too few pions.] Implicit in the above argument is the assumption that adding compressional energy in this fashion was a measure of the maximum number of pions is that the pion number was constant after it reached a maximum. However, as the system expanded and cooled, one might expect some of the pions to be absorbed. Besides this conceptual difficulty with the argument of Stock *et al.*, a newer intranuclear cascade model,[67] that includes binding energy can apparently account for the number of produced pions.

The calculations in both the hydrodynamic model as well as the intranuclear cascade model assumed the properties of the pion in the hot, dense matter were the same as for a free pion. This is known to be a poor approximation. The dispersion relation that describes the energy, ω, of a pion as a function of its momentum, $\mathbf{k}$, can be written as[61,68]

$$\omega^2 = \mu^2 + k^2 + \Pi(\mathbf{k},\omega) \; , \qquad (15)$$

where μ is the pion mass and $\Pi(\mathbf{k},\omega)$ is the polarization operator that depends on density and temperature as well as momentum and energy. In general, the polarization functions can be calculated in terms of Lindhard functions. However, a functional form was obtained by Migdal based on calculations by him and collaborators. Friedman *et al.*[68] modified this slightly in their initial work on the effects of the softening of the pion spectrum in heavy ion reactions.

$$\Pi(\mathbf{k},\omega) = \frac{k^2\Lambda^2\chi(\mathbf{k},\omega)}{1-g'\Lambda^2\chi(\mathbf{k},\omega)} \; ,$$

where $\Lambda = \Lambda(\mathbf{k})$ is a cutoff function defined by

$$\Lambda(\mathbf{k}) = e^{-\frac{k^2}{b^2}} \; , \quad b = 7m_\pi$$

and χ is the polarizability defined by

$$\chi(\mathbf{k},\omega) = \chi_R(\mathbf{k},\omega) + \chi_{ph}(\mathbf{k},\omega)$$

$$\chi_R(\mathbf{k},\omega) = -\frac{4\,a\,\omega_R\,\rho}{\omega_R^2 - \omega^2}\,, \qquad a = 1.13\; m_\pi^{-2}$$

$$\omega_R = m_\Delta + \frac{k^2}{2m_\Delta} - m_N$$

The quantity χ_R arises from the delta, ρ is the nuclear density, g' is the Landau parameter and ω_R is the energy of a Δ particle. [By fitting various scattering data, Johnson[69] has determined the value of g' to be 0.4±0.13.] The energy ω of a pion as a function of k is shown in Fig. 10; also plotted is ω_R and the energy ω_F of a free pion. For small k the excitations behave like free pions while for large k they behave like a Δ particle. Brown[70,71] has coined the term pisobars to describe the excitation. From the figure one may see that for intermediate values of k, $\omega(k)$ depends sensitively upon the value of g' and for sufficiently small g', one obtains a pion condensate.

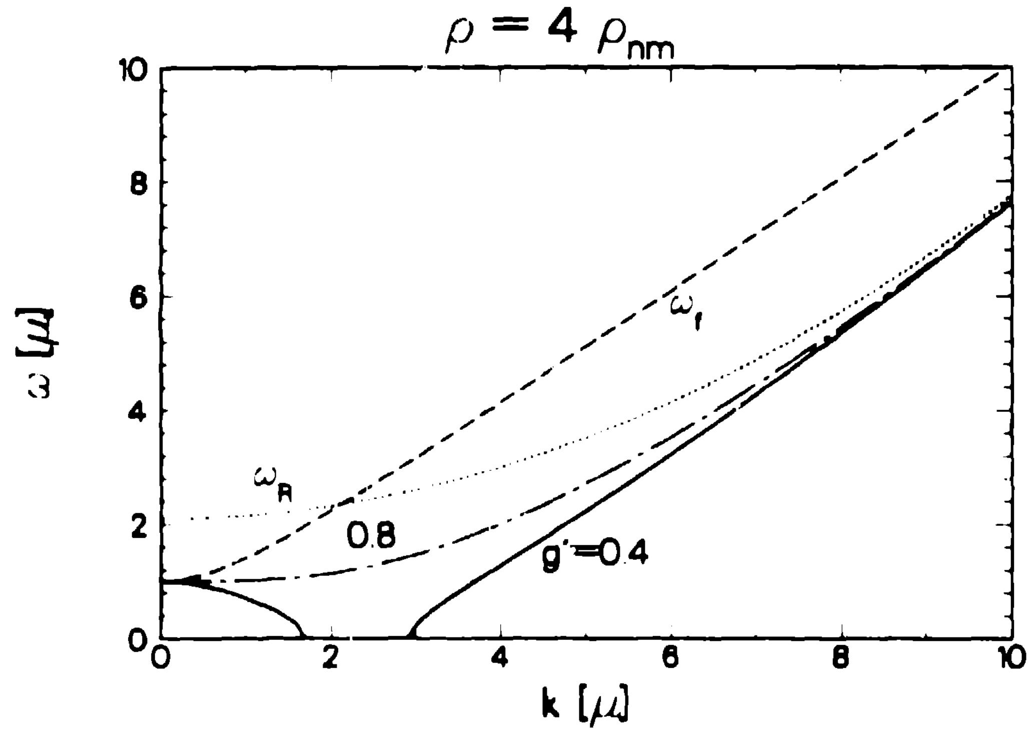

Fig. 10 The pion dispersion relation as defined in eqs. (3) and (4). The quantities ω_R and w_F are the energies of a Δ and a free pion, respectively.

For values of g' of 0.6 appreciably less energy is required to create a pion than were it to have the properties of a free pion. How does this affect the evolution of a heavy ion reaction and the number of pions that ultimately reach the detector? To determine this, a hydrodynamic calculation of a 800 MeV/nucleon equi-mass collision was performed. The only difference between the new calculation and that of Clare *et al*[65] is the change in the energy of the pion. In the hydrodynamic calculations one assumes there is both local thermodynamic equilibrium [*ie*, the concept of temperature is well-defined] and chemical equilibrium [the number of nucleons and pions have reached a steady state]. For a given temperature T and energy $\omega(k)$, the number of pions is

$$n_\pi = \frac{2}{\pi^2}\int dk\, \frac{k^2}{e^{\beta\omega(k)} - 1}$$

where $\beta = 1/kT$. The first effect of introducing pions is clear, with more degrees of freedom possible, the temperature of the system will be lower, the entropy higher and the observed ratio of neutrons to deuterons will be altered. In Fig. 14 are shown the calculated results for average temperature and number of pions as a function of time and value of g'. Values of g' less than 0.7 were not used as a pion condensate would be generated and solutions of the hydro equations would become singular. [These calculations assumed the number of pions was determined solely by the above equations; in actuality one must be more careful: the number of Δ's should not exceed the number of nucleons. This is not a problem except for small values of g'.] During the period of maximum compression, the temperature of the system is much less. This implies the thermal pressure will be less and the explosive expansion of the system will be slower.

In this calculation the system was allowed to expand indefinitely. However, at some value of the density hydrodynamics ceases to be valid and the system freezes out. By this, one assumes following freeze-out, the particles cease to interact and are free streaming. If, as in this calculation, one does not allow the pions to escape, one observes from the figure that the final number of pions will be the same, independent of the value of g'. Although this is an unrealistic assumption, one might inquire whether there is some methodwhereby one could determine the number of pions at the intermediate times. A cunning idea due to Gale and Kapusta [72,73] may allow this. They propose as a signature of the number of pions,the number of di-leptons produced during heavy ion reactions. The di-leptons are produced in the process

$$\pi\,\pi \rightarrow \gamma + \gamma\ .$$

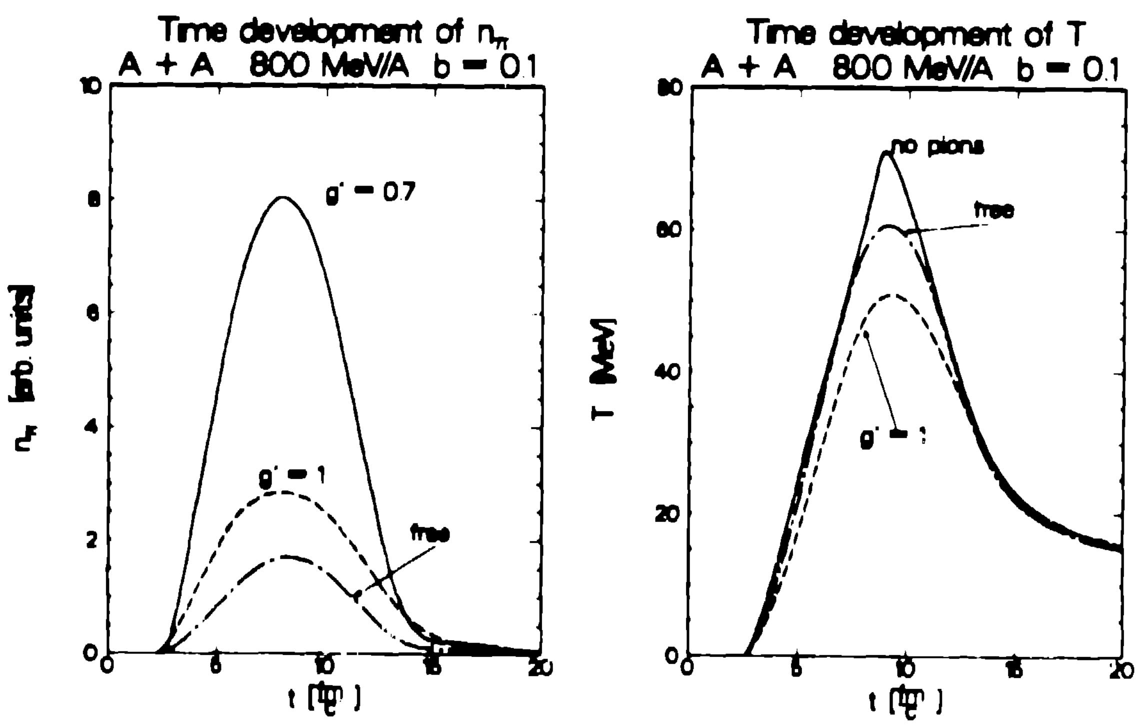

Fig. 11 Plot of T vs. t and n_π vs. t for a variety of assumptions of g'. The reaction was for equi-mass nuclei in a nearly central collision at 800 MeV per nucleon. The unit of time is fm/c, approximately 3 fm/c equals 10^{-23}s

However, theoretical results reported by Oset in these proceeding cast doubts on the possibility of observing an excess number of leptons. Experiments[74] that attempt to measure this are underway at the Bevalac. Thus, we see the close interplay between pion physics and heavy ion physics.

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